From Generative Collapse to Invariant Degrees of Freedom:

A Layered Closure Principle for Emergent Structure

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Abstract

Across mathematics and physics, structure is often described either as generated from primitive operations or as selected under constraint. These perspectives are typically treated independently, leaving open the question of how emergent structure participates in further generation.

In this work, we propose a unifying principle governing the transformation of degrees of freedom across layers of description. We show that generative systems under constraint produce invariant residual structures that, while insufficient to reconstruct their generating dynamics, retain enough information to serve as the effective generative basis for subsequent layers.

We formalize this as a *Layered Invariant Generator Principle*, in which degrees of freedom undergo a transformation from primitive generative variables to invariant structure, and are then reinterpreted as effective primitives at higher scales. This induces a recursive hierarchy in which generation, selection, and emergence form a closed cycle.

Within this framework, physical and mathematical horizons are reinterpreted as boundaries of invariant transport, marking the limits beyond which a descriptive layer can no longer sustain the structural information required to represent its generator. This provides a unified perspective on scale-dependent structure, the breakdown of physical theories, and the emergence of effective laws.

1 Introduction

Many successful frameworks across mathematics and physics exhibit a common structural pattern: a space of possibilities is constrained, and a subset of stable configurations persists. This appears in diverse forms, including renormalization group flow, decoherence, algebraic closure, and dynamical systems.

At the same time, independent lines of work demonstrate that complex systems may be generated from minimal primitives. Examples include universal logical gates, minimal combinatory systems, and recently, single-operator constructions capable of generating large classes of functions.

Despite these developments, a key structural question remains unresolved:

> How does emergent structure participate in further generation?

This question is central to understanding how physical laws, mathematical structures, and descriptive frameworks remain effective across scales despite the loss of underlying generative detail.

Existing frameworks typically describe either:

- generation from primitive rules, or

- selection and stabilization under constraint,

but do not close the loop between them.

The aim of this paper is to identify and formalize a closure principle governing the transformation of structure across layers. We show that emergent invariant structure, while not sufficient to reconstruct its generating dynamics, can function as the generative basis for subsequent layers.

2 Two Mechanisms of Structure Formation

We begin by distinguishing two fundamental mechanisms through which structure arises.

2.1 Generation by Completion

In many mathematical settings, structure is produced by freely adjoining operations or elements until closure conditions are satisfied. Examples include algebraic closure, free constructions, and combinatory systems.

These constructions exhibit:

- expansion of the configuration space,
- closure under specified operations,
- stable objects defined as those already closed under generation.

2.2 Selection by Stabilization

In contrast, physical and dynamical systems often exhibit structure through iterative constraint and elimination. In such systems:

- admissible transformations act on configurations,
- unstable configurations are suppressed,
- stable configurations persist as invariant structure.

This process may be formalized through idempotent operators or asymptotic stabilization, where repeated application of admissible dynamics yields a reduced invariant sector.

2.3 Structural Duality

These two mechanisms are structurally complementary:

- generation adds structure,
- selection removes instability.

However, they are not reducible to one another. In particular, selection-induced structure depends on the admissible dynamics that produce it, and is not uniquely determined by its stable outcomes.

3 Transformation of Degrees of Freedom

We now introduce a classification of degrees of freedom across layers. We use Σ_n to denote configuration spaces and G_n to denote their corresponding generative structure.

3.1 Primitive Generative Degrees of Freedom

At the most fundamental level, a system possesses a space of generative degrees of freedom:

$$\Sigma,$$

representing all admissible configurations prior to constraint.

3.2 Invariant Degrees of Freedom

Under repeated application of admissible dynamics, configurations are mapped into a reduced set:

$$I = \{x \in \Sigma \mid C(x) = x\},$$

where C denotes a stabilization or collapse operator.

These invariant degrees of freedom represent:

- persistence under constraint,
- stability under iteration,
- equivalence classes under admissible dynamics.

3.3 Effective Generative Degrees of Freedom

At the level of description, invariant structures are reinterpreted as primitive elements for a new layer. These form a new space:

$$\Sigma' \sim I,$$

which serves as the generative basis for higher-level dynamics.

Thus, degrees of freedom undergo a transformation:

$$\text{Primitive DOF} \to \text{Invariant DOF} \to \text{Effective Primitive DOF}.$$

4 The Layered Invariant Generator Principle

We now formalize the central claim.

> **Layered Invariant Generator Principle.** A generative system under constraint produces an invariant structure that:
>
> 1. does not uniquely determine the generating dynamics, and
> 2. retains sufficient structure to generate the next descriptive layer.

4.1 Non-Reconstructibility

The mapping from generative configurations to invariant structure is generally many-to-one. Distinct admissible dynamics may yield identical invariant sectors. As a result:

- the emergent layer cannot fully reconstruct its generator,
- information about eliminated configurations is irrecoverably lost.

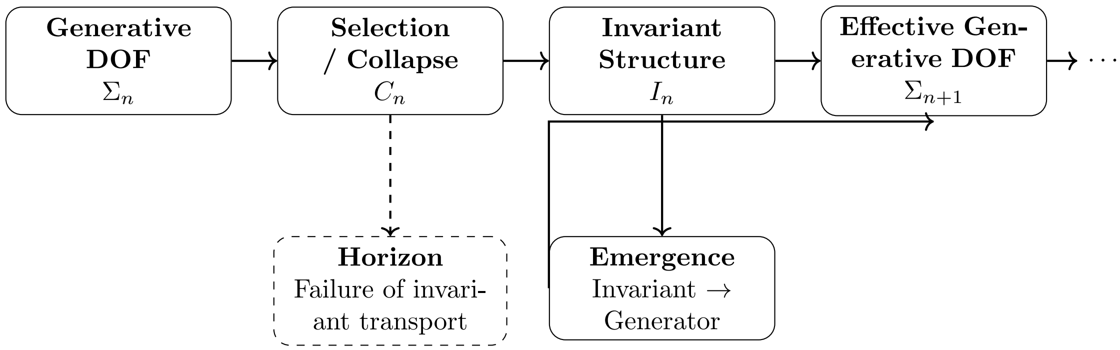

Figure 1: Layered transformation of degrees of freedom. Primitive generative degrees of freedom Σ_n are reduced under selection dynamics C_n to an invariant sector I_n. While I_n does not uniquely determine the underlying generative dynamics, it retains sufficient structure to serve as the effective generative basis Σ_{n+1} for the next layer. Horizons arise when invariant structure cannot be coherently transported to the next layer, forcing a transition in descriptive regime.

4.2 Invariant Sufficiency

Despite this loss, invariant structure encodes:

- persistent relations,
- symmetry constraints,
- stability conditions.

These are sufficient to define a new generative grammar at the next layer.

5 Closure Through Iteration

The transformation described above induces a recursive structure.

Let:

- G_n denote the generative structure at layer n,
- C_n denote the corresponding selection dynamics,
- I_n denote the invariant structure.

Then:

$$G_n \to C_n \to I_n \to G_{n+1}.$$

This defines a hierarchy in which:

- generation produces structure,
- selection stabilizes it,
- invariants form the basis of the next generator.

Thus, generation and selection form a closed cycle across layers.

Layered transformation of degrees of freedom. Primitive generative degrees of freedom Σ_n are reduced under selection dynamics C_n to an invariant sector I_n. While I_n does not uniquely determine the underlying generative dynamics, it retains sufficient structure to serve as the effective generative basis Σ_{n+1} for the next layer. Horizons arise when invariant structure cannot be coherently transported to the next layer, forcing a transition in descriptive regime.

5.1 Formalization of Layered Structure

We now formalize the transformation illustrated in Fig. 1.

Proposition 1 (Layered Invariant Generator Principle). *Let Σ_n be a space of generative degrees of freedom at layer n, and let*

$$C_n : \Sigma_n \to \Sigma_n$$

be a stabilization operator induced by a class of admissible dynamics. Define the invariant sector

$$I_n = \{x \in \Sigma_n \mid C_n(x) = x\}.$$

Then:

1. ***(Non-reconstructibility)** The invariant sector I_n does not, in general, uniquely determine the admissible dynamics that induce C_n.*
2. ***(Invariant sufficiency)** There exists a representation of I_n as an effective generative space Σ_{n+1} such that admissible transformations at layer $n+1$ can be expressed entirely in terms of structures internal to I_n.*
3. ***(Layered closure)** The transformation*

$$\Sigma_n \xrightarrow{C_n} I_n \sim \Sigma_{n+1}$$

defines a recursive structure in which invariant degrees of freedom at one layer become generative degrees of freedom at the next.

This principle may be interpreted as a structural closure condition on generative ontologies: while generative dynamics are not preserved across layers, invariant structure is sufficient to induce a new generative regime.

remark 1 (Relation to Fig. 1). Figure 1 illustrates the structure of Proposition 1. The forward flow

$$\Sigma_n \to C_n \to I_n$$

represents reduction under admissible dynamics, while the transition

$$I_n \to \Sigma_{n+1}$$

represents reinterpretation of invariant structure as effective generative degrees of freedom. The dashed region corresponds to regimes in which this transition fails, giving rise to horizon phenomena.

Proof sketch. (1) Non-reconstructibility follows from the fact that stabilization operators arise from admissible dynamics rather than universal constructions. Distinct admissible families may induce equivalent invariant sectors, implying that I_n does not uniquely encode the generating dynamics.

(2) Invariant sufficiency follows from the observation that I_n is closed under C_n and stable under admissible transformations. Relations among elements of I_n define a reduced structure that can be treated as primitive at the next layer, yielding an effective configuration space Σ_{n+1}.

(3) The recursive structure follows by iteration: applying admissible dynamics to Σ_{n+1} produces a further invariant sector I_{n+1}, which may again be reinterpreted as a generative space. This defines a hierarchy of layers connected by stabilization and reinterpretation.

The breakdown of this recursion corresponds to failure of invariant transport, as depicted in Fig. 1. □

6 Horizons as Boundaries of Invariant Transport

We now interpret physical and mathematical horizons within this framework.

Proposition 2 (Horizons as failures of invariant transport)**.** *Let Σ_n be a generative space at layer n, let*

$$C_n : \Sigma_n \to \Sigma_n$$

be a stabilization operator induced by admissible dynamics, and let

$$I_n = \{x \in \Sigma_n \mid C_n(x) = x\}$$

be the corresponding invariant sector.

Suppose that passage to the next layer requires a representation map

$$T_n : I_n \to \Sigma_{n+1}$$

which interprets invariant structure at layer n as effective generative degrees of freedom at layer $n+1$.

A horizon occurs whenever T_n fails to preserve the structural relations required for coherent generation at layer $n+1$. Equivalently, a horizon is present when one or more of the following conditions hold:

1. ***(Insufficient invariance)** the invariant sector I_n does not retain enough stable relational structure to define a nontrivial effective generator;*

2. ***(Loss of transport)** the map T_n is not well-defined on admissible equivalence classes;*

3. ***(Breakdown of closure)** the image $T_n(I_n)$ is not closed under the admissible dynamics of layer $n+1$.*

In such cases, the recursion

$$\Sigma_n \xrightarrow{C_n} I_n \xrightarrow{T_n} \Sigma_{n+1}$$

fails, and the descriptive regime at layer n reaches a structural boundary.

remark 2 (Horizon interpretation). Proposition 2 formalizes the dashed region in Fig. 1. A horizon does not imply failure of the underlying dynamics, but failure of a given layer to transport sufficient invariant structure into the next generative regime. In this sense, horizons are boundaries of invariant transport rather than boundaries of existence.

Proof sketch. The transition from I_n to Σ_{n+1} requires that invariant structure at layer n be sufficiently stable, distinguishable, and internally coherent to support a new generative description.

If I_n is too degenerate, then no nontrivial effective generator can be defined, yielding condition (1). If admissibly equivalent configurations in I_n are mapped inconsistently by T_n, then the next layer depends on distinctions that are not preserved under stabilization, yielding condition (2). If the image $T_n(I_n)$ fails to remain stable under the admissible transformations of layer $n+1$, then the induced layer is not dynamically closed, yielding condition (3).

In each case, invariant structure fails to propagate coherently across layers. The recursion therefore terminates or undergoes regime change. This structural failure is what we identify as a horizon. $\square$

In physical settings, this failure may appear as breakdown of geometric description, loss of coherence, or collapse of global coordination, depending on which invariant structures cease to propagate.

6.1 Examples of Horizon Formation

We briefly illustrate the three failure modes of Proposition 2 in representative physical settings. These examples are not exhaustive, but illustrate that horizon phenomena across scales may be understood as manifestations of the same underlying structural constraint.

Planck boundary (insufficient invariance). At sufficiently small scales, conventional geometric structure ceases to remain invariant under admissible dynamics. Metric relations, locality, and continuous degrees of freedom are no longer stable descriptors of the underlying configuration space. In the language of Proposition 2, the invariant sector I_n lacks the structural richness required to define a coherent geometric generator Σ_{n+1}. This corresponds to failure mode (1): insufficient invariance. The Planck scale may therefore be interpreted not as a fundamental cutoff, but as the boundary at which geometric invariants fail to persist under collapse, preventing further extension of spacetime-based description.

Decoherence and the quantum–classical boundary (loss of transport). In quantum systems, coherent phase relations define the invariant structure at the microscopic level. Under interaction with an environment, collapse dynamics suppress these relations, yielding classical observables as stable invariant sectors. However, the mapping from quantum invariants to classical variables is not invertible: distinct coherent configurations may collapse to identical classical states. This reflects failure mode (2): loss of transport. The representation map T_n from quantum invariant structure to classical degrees of freedom does not preserve all distinctions required to reconstruct the generator. Decoherence thus defines a horizon across which coherent phase information cannot be transported into the classical regime.

Cosmological horizon (breakdown of closure). At large scales, global geometric and causal structure depends on sustained coordination across distant regions. As separation increases, coupling strength and coherence degrade, limiting the ability of relational structure to remain dynamically closed. In this regime, local geometric descriptions remain valid, but cannot be consistently extended to a global generator. This corresponds to failure mode (3): breakdown of closure. The image $T_n(I_n)$ fails to remain stable under the admissible dynamics required for global coordination, producing a horizon beyond which a single coherent geometric description cannot be maintained.

7 Relation to Renormalization Group

The layered transformation of degrees of freedom described in this work bears a close structural relationship to renormalization group (RG) flow, while differing in interpretation and scope.

7.1 RG as Invariant Degree-of-Freedom Transformation

In RG theory, a system is described at successive scales by progressively eliminating or coarse-graining microscopic degrees of freedom. This process induces a flow on the space of effective theories:

$$\mathcal{T}_\Lambda \to \mathcal{T}_{\Lambda'}$$

where $\Lambda' < \Lambda$ denotes a lower-resolution scale.

Under RG flow:

- microscopic degrees of freedom are integrated out,
- irrelevant operators are suppressed,
- relevant and marginal operators persist,
- fixed points define scale-invariant structure.

From the perspective of the present framework, this corresponds precisely to a transformation:

$$\Sigma_n \xrightarrow{C_n} I_n$$

in which:

- Σ_n represents the full configuration space at a given scale,
- C_n corresponds to coarse-graining or integration over short-scale structure,
- I_n corresponds to the invariant sector defined by RG-relevant structure.

Thus, RG flow may be interpreted as a concrete realization of invariant degree-of-freedom reduction.

7.2 Emergent Generators and Fixed Points

A central feature of RG is the emergence of effective theories at different scales, governed by a reduced set of parameters. These effective descriptions are not derived from first principles at each scale, but instead arise from the persistence of certain structures under RG flow.

In the present framework, this corresponds to the reinterpretation:

$$I_n \sim \Sigma_{n+1}$$

in which RG-invariant or RG-relevant structures become the effective generative degrees of freedom at the next layer.

Fixed points of RG flow play a special role: they correspond to structures that are invariant under repeated coarse-graining. These may be identified with particularly stable invariant sectors:

$$C_n(I_n) = I_n$$

which serve as attractors in the space of descriptions.

7.3 RG as a Special Case of Layered Closure

While RG provides a well-developed formalism for scale transformation in physical systems, it operates within a specific class of models and assumes a predefined notion of scale and coarse-graining.

By contrast, the Layered Invariant Generator Principle applies more generally:

- to arbitrary admissible dynamics, not only scale-based coarse-graining,
- to abstract configuration spaces beyond field-theoretic models,
- to transformations where "scale" is replaced by constraints on invariance and admissibility.

From this perspective, RG may be understood as a special case of the broader transformation:

$$\Sigma_n \to C_n \to I_n \to \Sigma_{n+1}$$

in which:

- coarse-graining implements C_n,
- RG-relevant operators define I_n,
- effective field theories define Σ_{n+1}.

7.4 Horizons and RG Breakdown

Within this interpretation, the breakdown of RG description corresponds to horizon formation in the sense of Proposition 2.

Specifically:

- when no stable fixed point exists, invariant structure fails to persist (insufficient invariance),
- when coarse-graining loses essential distinctions, effective variables cannot represent underlying structure (loss of transport),
- when effective theories fail to remain closed under scale transformation, RG flow becomes inconsistent (breakdown of closure).

Thus, the limits of RG applicability may be interpreted as boundaries of invariant transport, beyond which a given scale-dependent description ceases to function as a coherent generator.

7.5 Interpretation

This correspondence suggests that renormalization group flow is not merely a technical tool, but an instance of a more general structural principle:

> Renormalization group transformations describe how invariant degrees of freedom emerge under constraint and become the effective generators of higher-level description.

In this sense, RG provides a concrete realization of the layered closure of generative structure described in this work.

This perspective may be further refined by introducing a coordination capacity functional $C[R]$, defined over relational configurations, which measures the ability of a system to sustain invariant structure under constraint; horizon formation then corresponds to regimes in which $C[R]$ falls below the threshold required to support coherent invariant transport across layers.

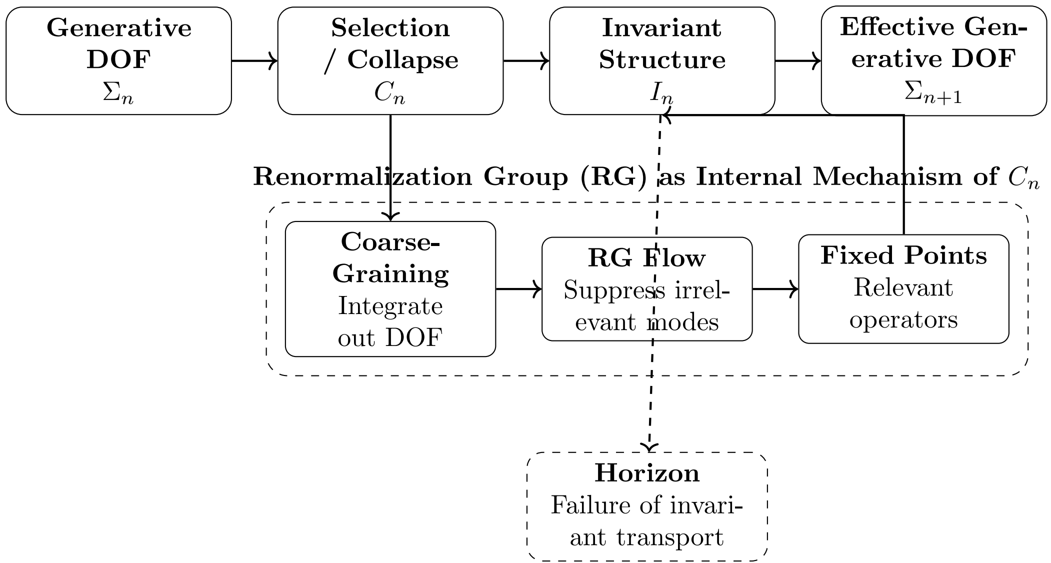

Figure 2: **Renormalization group flow within the layered transformation of degrees of freedom.** The collapse/selection operator C_n may be instantiated by renormalization group dynamics, in which coarse-graining eliminates microscopic degrees of freedom and suppresses irrelevant operators. The resulting fixed-point structure corresponds to the invariant sector I_n, which is then reinterpreted as the effective generative degrees of freedom Σ_{n+1}. Horizons arise when this invariant structure fails to support coherent generation at the next layer.

8 Implications

8.1 Emergent Laws as Compressed Generators

Effective laws at higher levels are not fundamental primitives, but invariant-compressed generators derived from lower-level dynamics.

8.2 Limits of Unification

Attempts to reconstruct a generating layer from its emergent invariants are structurally limited. This explains the persistence of incomplete or scale-dependent theories.

8.3 Layered Ontology

Reality is organized not as a single generative system, but as a hierarchy of layers connected by invariant transformation and re-generation.

9 Conclusion

We have proposed a general principle governing the transformation of structure across layers of description. Generative systems under constraint produce invariant degrees of freedom that, while insufficient to reconstruct their origin, serve as the basis for further generation. This framework suggests that the emergence of effective laws, physical horizons, and scale-dependent structure may be governed by a common underlying transformation of degrees of freedom, providing a basis for further formal and empirical investigation.

This establishes a closed architecture in which:

- structure is generated,
- stabilized under constraint,
- compressed into invariants,
- and re-expanded as new generative structure.

Within this framework, horizons emerge as boundaries of invariant transport, and effective laws arise as compressed representations of deeper dynamics.

The result is a layered ontology in which generation and selection are not competing mechanisms, but complementary phases of a recursive process.